



**The Hong Kong University of Science and Technology**

**Department of Mathematics**

**PHD THESIS EXAMINATION**

*Large solutions to fractional  $Q$ -curvature equations*

*By*

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***ABSTRACT***

In this thesis, we study positive solutions to the fractional  $s$  order  $Q$ -curvature equation

$$(-\Delta)^s u = K(x)u^{\frac{n+2s}{n-2s}},$$

where  $s \in (0, n/2)$ . When  $s \in \mathbb{N}_+$  and  $K \equiv 1$ , we prove an upper blow up rate and asymptotic symmetry of the singular solutions near the singular set. When  $s \in (1/2, 1)$ , in lower dimensions, we show that for any positive  $C^1$  function  $K$ , a singular solution  $u$  satisfies an upper blow up rate near the origin. In contrast, when  $s \in (0, 1)$  or  $s \in \mathbb{N}_+$ , and  $n > 2s + 3$ , we construct a positive  $C^1$  function  $K$  such that its singular solution  $u$  can be arbitrarily large near the origin. When  $s \in (0, 1)$  and  $K$  is negative in  $B_2$ , we construct a sequence of solutions that blows up in  $B_1$ , which is a different phenomenon from the classical Nirenberg problem.

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*(Open to all faculty and students)*

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).